Neutrino Decay

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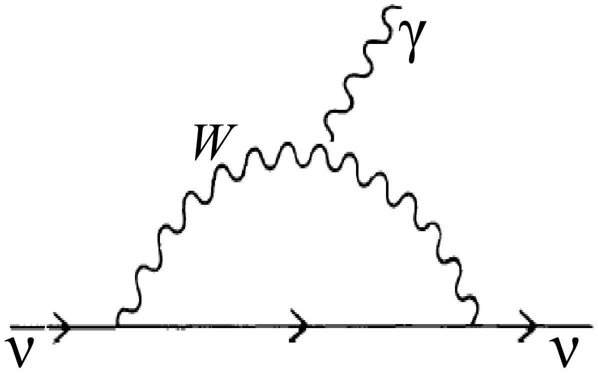
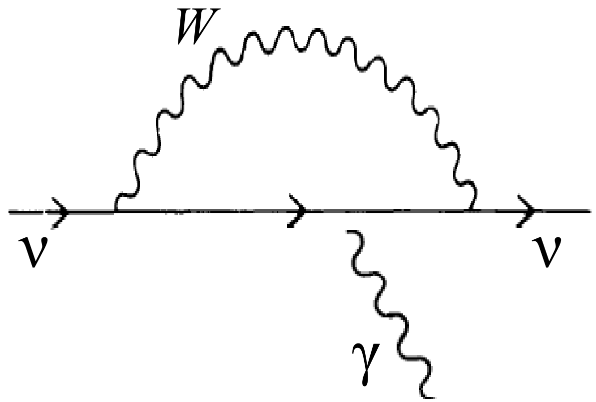
Outline

1. Decay Process
2. Dirac v.s. Majorana
3. Electromagnetic Properties
4. Majorana Quantization
5. Calculation

Decay Process

$$\nu_H \rightarrow \nu_L + \gamma$$





$$\nu_H \rightarrow \nu_L + \nu'_L + \nu'_L$$

Dirac v.s. Majorana

For **Dirac** particle

Mass term $\sim \bar{\psi}_R \psi_L$

If only left-hand neutrino exists, **Majorana** mass is the only possible way to gain the mass.

For the Majorana condition: $\psi = \psi_C$,

the requirement of complex components: **4 → 2**

Electromagnetic Properties

$$< \psi(p')|J_\mu^{EM}|\psi(p) >= \bar{u}(p')\Lambda_\mu(q,l)u(p)$$

$$q_\mu = p'_\mu - p_\mu, \; l_\mu = p'_\mu + p_\mu$$

$$\Lambda_\mu(q) = f_Q(q^2)\gamma_\mu + f_M(q^2) i\sigma_{\mu\nu}q^\nu + f_E(q^2)\sigma_{\mu\nu}q^\nu\gamma_5 + f_A(q^2)(q^2\gamma_\mu - q_\mu \not{q})\gamma_5$$

f_Q, f_M, f_E, and f_A are form factors indicating the properties of neutrino.

In the Dirac neutrino case, for the assumption of CP invariance contained in the electromagnetic current operator gives the zero f_M. And in Majorana case, f_Q, f_M, f_E vanish, regardless of the CP invariance.

Since Dirac and Majorana neutrino exhibit quite different properties, it improve us to investigate the electromagnetic interaction of neutrinos.

Upper bound to neutrino charge: $q_\nu \leq 10^{-21} e$

The non-trivial electromagnetic properties imply the "**direct**" coupling between photon and neutrino.
And it makes several interactions are possible:

1. $\nu_1 \rightarrow \nu_2 + \gamma$

2. $\gamma \rightarrow \nu\bar{\nu}$

3. neutrino scattering of electron(or nuclei)

4. neutrino spin procession in magnetic field

Quantization of Majorana Field

Gamma matrices in **Dirac representation**:

$$\gamma^0 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} , \quad \gamma^i = \begin{pmatrix} 0 & \sigma^i \\ -\sigma^i & 0 \end{pmatrix}$$

$$\gamma^5 = i\gamma^0\gamma^1\gamma^2\gamma^3 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

In **Weyl representation**:

$$\gamma^0 = \begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix} , \quad \gamma^i = \begin{pmatrix} 0 & \sigma^i \\ -\sigma^i & 0 \end{pmatrix}$$

$$\gamma^5 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

Use the projection operators separate the field into left-hand and right-hand:

$$\psi_L \equiv \frac{1-\gamma_5}{2}\psi = \begin{pmatrix} 0 \\ \eta \end{pmatrix} \qquad \psi_R \equiv \frac{1+\gamma_5}{2}\psi = \begin{pmatrix} \xi \\ 0 \end{pmatrix}$$

The charge conjugate:

$$\psi^c = i\gamma^2\psi^*$$

$$(\psi_L)^c = \begin{pmatrix} i\sigma_2\eta^* \\ 0 \end{pmatrix} = \frac{1+\gamma_5}{2}\begin{pmatrix} i\sigma_2\eta^* \\ 0 \end{pmatrix}$$

$$(\psi_R)^c = \begin{pmatrix} 0 \\ -i\sigma_2\xi^* \end{pmatrix} = \frac{1-\gamma_5}{2}\begin{pmatrix} 0 \\ -i\sigma_2\xi^* \end{pmatrix}$$

Under the Weyl representation, the Lorentz transformation is reduced into two 2 × 2 matrices.

$$D(\alpha) = \begin{bmatrix} \exp[-i(\frac{i}{2}\alpha_{0i}\sigma_i + \frac{\alpha_{ij}}{2}\epsilon_{ijk}\sigma_k)] & 0 \\ 0 & \exp[-i(-\frac{i}{2}\alpha_{0i}\sigma_i + \frac{\alpha_{ij}}{2}\epsilon_{ijk}\sigma_k)] \end{bmatrix}$$

$$\eta(x) \to \eta'(x') = M\eta(x) = \exp\left[-i\left(\frac{\alpha_{ij}}{2}\epsilon_{ijk}\sigma_k - i\frac{\alpha_{0j}}{2}\sigma_j\right)\right]\eta(x)$$

$$\xi(x) \to \xi'(x') = \tilde{M}\xi(x) = \exp\left[-i\left(\frac{\alpha_{ij}}{2}\epsilon_{ijk}\sigma_k + \frac{\alpha_{0j}}{2}\sigma_j\right)\right]\xi(x)$$

Majorana condition for Majorana field χ :

$$\chi^c = \pm\chi$$

It's obvious that $\chi \equiv \psi_L + \psi_L^c$ satisfies the condition.
The Lagrangian with mass m is given by

$$\mathcal{L} = \frac{i}{2}\bar{\chi}\,\partial\!\!\!/\chi - \frac{1}{2}m\bar{\chi}\chi$$

A more general field can be expressed as

$$\chi = (e^{i\alpha}\psi_L + e^{i\beta}\psi_L^c)$$

And its charge conjugate:

$$\chi^c = e^{-i(\alpha+\beta)}\chi$$

We rewrite the Lagrangain:

$$\mathcal{L}' = i\bar{\psi}_L \, \partial\!\!\!/ \psi_L - \frac{m}{2}\left[e^{-i(\alpha-\beta)}\bar{\psi}_L \psi_L^c + e^{i(\alpha-\beta)}\bar{\psi}_L^c \psi_L\right]$$

Writing

$$M = me^{i(\alpha-\beta)}$$

We get

$$\mathcal{L}' = i\bar{\psi}_L \, \partial\!\!\!/ \psi_L - \frac{M}{2}\bar{\psi}_L^c \psi_L + \text{h.c.}$$

Although the mass *M* has a complex phase, it can be absorbed by ψ_L .

The mass term implies the breaking of lepton number conservation.

Here we transform the Lagrangian into 2-column form, rewriting it in terms of the Weyl spinor field η .

$$\mathcal{L} = i\eta^\dagger(\sigma^\mu \partial_\mu)\eta - \frac{i}{2}m\eta^T\sigma_2\eta + \frac{i}{2}m\eta^\dagger\sigma_2\eta^*$$

The Euler-Lagrange equations:

$$\sigma^\mu \partial_\mu \eta + m\sigma_2\eta^* = 0$$
$$\tilde{\sigma}^\mu \sigma_2 \partial_\mu \eta^* - m\eta = 0$$

where

$$\sigma^\mu \equiv (1, -\sigma_i) \qquad \tilde{\sigma}^\mu = (1, \sigma_i)$$

Combining two equations, and we obtain

$$(\partial_0^2 - \nabla^2 + m^2)\eta = 0$$

Just the same as the quantization as Dirac fields, firstly we find the canonical conjugate field $\pi(x)$. And write down the anticommuation relation.

$$\{\eta(\mathbf{x},t),\pi(\mathbf{x}',t)\} = i\delta(\mathbf{x}-\mathbf{x}') ,$$
$$\{\eta(\mathbf{x},t),\eta(\mathbf{x}',t)\} = \{\eta^*(\mathbf{x},t),\eta^*(\mathbf{x}',t)\} = 0$$

We expand the $\eta(x)$ as

$$\eta(x) = \frac{1}{(2\pi)^{3/2}}\int d^3p\sqrt{\frac{p_0+|\mathbf{p}|}{2p_0}}\{[a_-(\mathbf{p})\beta(\mathbf{p})+b_+(\mathbf{p})\alpha(\mathbf{p})]e^{-ipx} + [c_-(\mathbf{p})\beta(\mathbf{p})+d_+(\mathbf{p})\alpha(\mathbf{p})]e^{ipx}\}$$

Where $\alpha(\mathbf{p})$ and $\beta(\mathbf{p})$ are helicity eigenspinors.

$$d_+ = \frac{m}{p_0+|p|}a_-^\dagger \ , \ b_+ = -\frac{m}{p_0+|p|}c_-^\dagger$$

Feynman rules

The propagator:

$$\langle 0|T[\eta(x)\eta^\dagger(x')]\ |0\rangle = \frac{1}{(2\pi)^4}\int d^4p\frac{\tilde{\sigma}^\mu p_\mu}{p^2-m^2}\ e^{-ip(x-x')}$$

$$\langle 0|T[\eta(x)\eta^T(x')]\ |0\rangle = \frac{1}{(2\pi)^4}\int d^4p\frac{im\sigma_2}{p^2-m^2}\ e^{-ip(x-x')}$$

The non-vanishing propagator indicates a violation of lepton number.

Calculation

Now we have the diagram and know the propagator, but we do not know the interaction $\mathcal{L}_{int} = ?$

The possible vertices and coupling constant are model dependent. However, for a simple decay, we can do some estimation.

An approximation is given by

$$\Gamma(\nu_H \rightarrow \nu_L + \gamma) \sim \alpha(\text{mixing factor}) \left[\frac{\text{charged}-\text{lepton mass}}{W-\text{meson mass}} \right]^4 \left| \frac{m(\nu_H)}{m(\mu)} \right|^5 \Gamma(\mu \rightarrow e \bar{\nu}_e \nu_\mu)$$

$$\sim 10^{-28} \epsilon^5 \mathrm{MeV}$$

$$\Gamma(\nu_H \rightarrow 3\nu_L) \sim \epsilon^4 \left| \frac{m(\nu_H)}{m(\mu)} \right|^5 \Gamma(\mu \rightarrow e + \bar{\nu}_e + \mu) \quad \sim 10^{-16} \epsilon^9 \mathrm{MeV}$$